1.1 Shirts and Caps: Solving Equations With Two Variables

Focus Question What kind of solutions will be found for an equation with two variables such as $3x + 5y = 13$? What will the graphs of those solutions look like?

Launch

Equations from previous Units are shown in the introduction to the Investigation. Review how to solve equations like these. Have students describe the features of the graphs of the equations.

Begin by describing the Problem. Ask students to describe in words how much profit the class will make from each item in their sale. Clarify the fact that *profit* and *selling price* are not the same.

Materials

Labsheets
- 1.1A: Fundraiser Sales
- 1.1B: Question C

Accessibility Labsheet
- 1ACE: Exercise 2

- graph paper
- graphing calculator (optional)
- Coordinate Grapher

Explore

The Questions of this Problem provide an initial exploratory environment with a story context. Then, students are pushed to think more generally about the mathematics by analyzing equations without context support.

Have students work on Questions A–C. You may want to bring the class together to discuss Question D. As students work on the parts of each Question, make sure that they understand what is being asked. Specifically in Question B, part (1), make sure they are producing numbers of T-shirts and caps that together produce the desired $600 profit. If students are unsure of their first step, encourage them to begin with one of the variables (either caps or T-shirts) and to make guesses with numbers that make for easy calculations, such as multiples of 10.

Summarize

The key objective of this Problem is to have students develop the understanding that equations in the form $Ax + By = C$ have, in general, infinitely many solutions that can be represented as ordered pairs (x, y). The graphs of those solution sets of ordered pairs will always be straight lines.

You can make the Focus Questions of this Problem the center of the Summarize. Along with each question within the Focus Question, ask:

- *What evidence do you have to support your answer?*

Revisit the graph for Question B at the beginning of the Summarize. Then, ask students to share the graphs they found for Question C.

- *What do your results suggest about the likely shape of the graph for solutions to any equation that looks like $Ax + By = C$?*

ACE

Assignment Guide for Problem 1.1

Applications: 1–8 | Connections: 28–35

Answers to Problem 1.1

A. **1.** $\$175 = \$5(15) + \$10(10)$

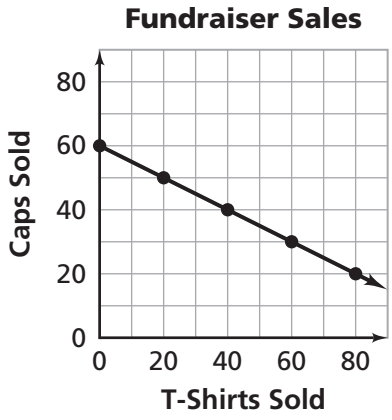
2. $\$260 = \$5(12) + \$10(20)$

3. $\$650 = \$5(30) + \$10(50)$

4. $5s + 10c = P$

B. **1.** There are many possible pairs of numbers which include (0, 60), (120, 0), (10, 55), (20, 50), and (30, 45).

2. The complete graph will look similar to this one.

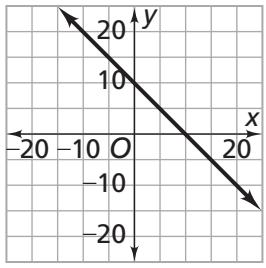


3. The linear pattern in students' five plotted points should suggest some other ordered pairs that satisfy the equation.

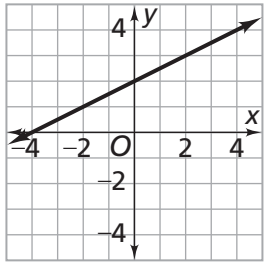
4. No, because the price of each item and the profit goal does not change so all possible solutions will stay the same. However, the answers will be in (*c*, *s*) form instead of (*s*, *c*) form, which changes the appearance of the graph.

C. In each case, there are many possible solutions. It might be a good idea to encourage students to develop the habit of looking for intercept solutions first.

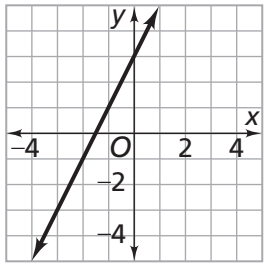
1. The solutions pairs include (0, 10), (10, 0), (1, 9), (−1, 11), and (−2, 12). The graph will also show pairs with decimals such as (0.5, 9.5) and (−3.5, 13.5).



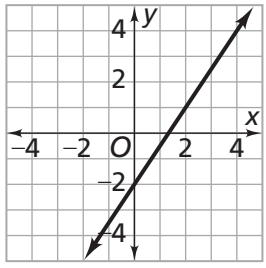
2. The solution pairs include (−4, 0), (−2, 1), (0, 2), (2, 3), and (4, 4). Decimal solutions include (1, 2.5) and (3, 3.5).



3. The solution pairs include (−1.5, 0), (0, 3), (1, 5), (−2, −1), and (−3, −3).



4. The solution pairs include $\left(\frac{4}{3}, 0\right)$, (0, −2), (2, 1), (−2, −5), and (−4, −8).



D. Students should have the idea that there are many solutions to such equations and that the graph of those solutions will be a straight line. To emphasize this idea, you might choose a point not on the line to show that its coordinates do not satisfy the given equation.